Sigma Metric

in Analytical Laboratory Performance

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Objectives

- Review of Normal distribution
- Concept of Sigma Metric
- Characteristics of Sigma Metric
- Applying Sigma Metric
- Short-term/Long-term issue in medical laboratory

Background

Six Sigma management technology:

- Developed by Motorola in the 1980s
- To systematically improve processes and eliminate defects
- Requiring to fall within plus or minus six sigma from the process mean

Sigma Metric:

- Core concept in Six Sigma Methodology
- Defect Rate; DPM or DPMO
- Based on Gaussian (Normal) distribution



Gaussian Distribution



Body Mass Index

- Mean = 23 Kg/m²
- SD = 1.5 Kg/m²



- Area under curve for x_i depends on the number of SDs from mean at x_i
- Irrespective of different Means and SDs, AUC for the same number of SD is the same.



Standard Gaussian Distribution

Z value: The position of a variable in terms of its distance from the mean when measured in stanDEVard deviation units.







Unacceptable parts = 16%



What & Why of SM?

Main aspects of laboratory analytical practice, i.e.

- Performance Specifications, Define Quality; DRa
- Method Evaluation, and <u>Measure Quality; Lab DR?</u>
- Quality Management, Quality Management, QC; Keep Lab DR<DRa</p>
- are about Defect Rate, and
- SM is a measure of Defect Rate

Where SM?

SM is a useful tool:

- •Assess the analytical quality of assays; Method Evaluation,
- Planning quality control strategies (IQC),
- Describe assay analytical performance in EQA
- Improve performance

Sigma Metric

What? A measure of Defects

Why? Defect rate is an indicator of Reliability

Where? In the all aspects of analytical performance

SPECIFICATION in Six Sigma

Target Value; TV



- 1. Allowable Deviation; DEVa: How much deviation from TV is acceptable?
 - Tolerance Limit, TL
 - Perfect/Defect

2. Allowable Defect Rate; DRa:

How many defects are acceptable?

DEVa vs DRa





1. DEVa: Acceptance criterion for a single product/occurrence/opportunity

Reliability

2. DRa: Acceptance criterion for performance





- Performance EVALUATION
- 1. Determine Defect Rate
- 2. Is Defect Rate ≤ Allowable Defect Rate?



Counting Approach



Probabilistic (Distribution-Based) Approach

Counting methodology





Probabilistic Approach

- Sampling from products
- Mean?
- Centered?
- Imprecision?
- > AUC at tails out of TLs?



Manufacturing metal balls

Target Dimeter = 100 mm

SPECIFICATION

DEVa = 7%

DRa = 5%

EVALUATION

- Mean = 100 mm
- SD = 3 mm

Calculation: Defect Rate?



Manufacturing metal balls

- Target Value = 100 mm
 - Mean = 100 mm

SD = 3 mm

- DEVa = 7%
- DRa = 1%



Manufacturing metal balls

Target Value = 100 mm

Mean = 100 mm

SD = ?

DEVa = 7%

DRa = 1%

Z@LTL = NORM.S.INV(0.005) = -2.6 Z@UTL = NORM.S.INV(1-0.005) = 2.6

$$z = \frac{x - \overline{X}}{SD} \rightarrow SD = 7 \text{ mm/2.6} = 2.7 \text{ mm}$$





 \succ Z at TL: Number σ s (SDs) between mean and TL

Z value of TLs is a measure of Defect Rate

 \checkmark The larger is Z at TL, the smaller is the tail beyond TL



WBC





Q: If Z of TL is measure of defect rate, Z at which TL is the right indicator of defect rate?





Centered vs Off-Center; Industrial origin

Capability Index

• No shift:

Cp = (UTL - LTL)/6SD



• Shift:

Cpk = min [(μ – LTL)/3SD, (UTL– μ)/3SD]





Concerning Defect Rate, we can call this a "3 σ " performance



Definition of SIGMA METRIC

- > Number of Sigmas (SDs) between Mean and the *nearest* TL;
- > Equals to Z of the nearest TL (Z of TL at the bias side)

Sigma Metric Equation

1. Number of SDs between Mean and the nearest TL



Sigma Metric Equation

2. Z value of the nearest TL to Meant



SM Equation; Adapted for analytical performance


Characteristics of Sigma Metric

- 1. SM and DR aren't linearly related
- 2. Numerical calculations (+, -, x, ÷) can be done on SM values
- 3. Zero SM means Bias = TEa and Short-term DR = 50%
- 4. Negative SM means Bias > TEa and Short-term DR > 50%
- 5. With biased performance, "(TEa-B)/SD" gives correct SM
- 6. Long-term DR is calculated by subtracting 1.5 from calculated SM
- 7. One-side calculation is used to determine long-term DR

1. Relation between SM and DR?



1. Relation between SM and DR?

NOTE 1: SM and DR aren't linearly related.





Earthquake amplitude and Richter scale aren't linearly related!



And also, immunoassay Signal/Concentration curves!









NOTE 2:

- Bias as multiples of SD can be added to- or subtracted from SM; and
- SM can be multiplied or divided by factors.







3. Zero Sigma?

NOTE 3: Zero SM doesn't mean 100% defects (or 0% yield)!

With a SM=0, DR is approximately 50% (and Yield ≈ 50%)



4. Negative Sigma?

NOTE 4: Negative yield is not meaningful, but negative SM is!

With a negative SM, DR is >50% (and yield is <50%).



Example 1:

- TV=100 mg/dL
- TEa = $\frac{8}{\text{mg/dL}}$
- Mean = 100 mg/dL
- Bias = 0 mg/dL
- SD = 2 mg/dL

TEa% = (TE/TV) x 100

B% = (B/TV) x 100

CV = (SD/Mean) x 100

SM = (TEa - B)/SD

SM = (8 - 0)/2 = 4

SM = (TEa% - B%)/CVSM = (8% - 0)/2% = 4

Example 2:

- TV=100
- TEa = 8
- Mean = 103
- Bias = 3
- SD = 2

TEa% = (TE/TV) x 100 B% = (B/TV) x 100 CV = (SD/Mean) x 100 SM = (TEa - B)/SDSM = (TEa% - B%)/CVSM = (8 - 3)/2 = 2.50SM = (8% - 3%)/1.96 = 2.58

Example 3:

- TV=100
- TEa = 8
- Mean = 97
- Bias = -3
- SD = 2

TEa% = (TE/TV) x 100 B% = (B/TV) x 100 CV = (SD/Mean) x 100 SM = (TEa - B)/SDSM = (TEa% - B%)/CVSM = (8 - 3)/2 = 2.50SM = (8% - 3%)/2.06 = 2.43

NOTE 5: When there is bias, (TEa - B)/SD and (TEa% - B%)/CV give a bit different SMs; First one correct.

TV	ТЕа	X	Bias	SD	CV	SM = (TEa-B)/SD	SM = (TEa%-B%)/CV
100	8	103	+3	2	1.94	2.50	2.58
100	8	97	-3	2	2.06	2.50	2.43

$$\frac{A-B}{C} = \frac{\frac{A}{D} - \frac{B}{D}}{\frac{C}{D}}$$

TEa% = TEa x 100/TV Bias% = Bias x 100/TV CV= SD x 100/X

$$\frac{\text{TEa} - B}{\text{SD}} \neq \frac{\frac{\text{TEa} \times 100}{\text{TV}} - \frac{B \times 100}{\text{TV}}}{\frac{\text{SD} \times 100}{\overline{X}}}$$
$$\frac{\frac{\text{TEa} - B}{\text{SD}} \neq \frac{\text{TEa}\% - B\%}{\text{CV}}$$

6. Defect Rate: SHORT-TERM vs. LONG-TERM?

		Short Term	Defects	
		Sigma Level	Per Million	
Example		6	3	
•		5.9	5	
• $TE_2 = 6$		5.8	9	
• ILa – 0		5.7	13	
		5.6	21	
• Blas = 2		5.5	32	
		5.4	48	
• SD = 1		5.3	72	
		5.2	108	
• $SNI - I$		5.1	159	
-3101 = 4		5	233	``
	4.9	337	71	
• $P(Z>4) = 32 DPIVI$	4.0	403	1.4	
		4.7	00	OO V
		4.0	13	
		4.5	1.8	
	P(Z > 2.5) = 6210	4.3	2 55	
l		4.2	3,467	
		41	4,661	
		4	6,210	>>>32
		3.9	8,198	
		3.8	10,724	
		3.7	13,903	
		3.6	17,864	
		3.5	22,750	

ANSWER

- Calculated SM is a *REPORT* about the **PAST!**
- > We need to *FORESEE/ASSURE* DR for the **FUTURE!**

Given:

- Usually SM is determined in Short-term evaluation
- The variable factors are not completely evaluated in Short-term
- QC strategies cannot detect small shifts

***** A higher DR is expected for Long-term

Defect Rate: SHORT-TERM vs. LONG-TERM?

- NOTE 6: In Six Sigma methodology, to calculate Long-term DR, 1.5 is subtracted from the SM calculated from Short-term DEVata.
 - > Assumptions in the Six Sigma methodology:
 - In long-term shifts of different sizes and in both directions happen
 - The largest expected shift (the worst case) is 1.5 SD
 - The shifts are not detectable and/or correctable
 - The shifts are reversed by themselves
 - By convention established at Motorola, the Sigma level is adjusted by 1.5 sigma to recognize the tendency of processes to shift over the long term

Long-term DR = P(Calculate SM - 1.5)

Calculated SM corresponds to two defect rates:

- Short-term DRWhat did happen?
- Long term What is expected?

Defects per Million	Sigma Level			
Opportunities	(With 1.5 Sigma Shift)*			
933193	0.000			
915434	0.125			
894350	0.250			
869705	0.375			
841345	0.500			
809213	0.625			
773373	0.750			
734014	0.875			
691462	1.000			
646170	1.125			
598706	1.250			
549738	1.375			
500000	1.500			

Calculated SM corresponds to two defect rates:

- Short-term DRWhat did happen?
- Long term What is expected?



Calculated SM corresponds to two defect rates:

- Short-term DRWhat did happen?
- Long term What is expected?



7. DR: One-side or Two-side probability?

- NOTE 7: For calculating long-term DR (e.g. presented in Sigma tables), one-side Gaussian probability is determined.
 - The long-term DR is calculated assuming 1.5
 - When bias>1 SD, the defects at TL away from bias is ignorable
 - For calculating long-term DR, only the tail beyond the TL at bias side matters

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Sigma Level or Defect Rate?







Example:

- ISO 15189:2012; 3-19
 - Quality Indicators: % yield, % defects, DPMO, Six Sigma scale

• IFCC-WG on A1C

- Least acceptable quality of A1C testing: **2 Sigma** for routine testing and **4 Sigma** for clinical trials

Applications of Sigma Metric

1. Goal Setting; Least acceptable SM, SMa

- In the TE model, 'TE = B + 2SD' corresponds to the least acceptable quality is 2 Sigma
- Also **'TE = B + 3SD'** and **'TE = B + 4SD'** are recommended that correspond to the least quality of **3 Sigma** and **4 Sigma**
- Motorola goal: 6 Sigma
- ISO 15189 proposes SM as QI
- Expert groups such as the IFCC-WG A1C set goals as least acceptable Sigma

Applications of Sigma Metric

2. Method Evaluation; Performance SM

• If SM \geq SMa \rightarrow Acceptable

Example; A1C

- SMa = 2 for routine and 4 for clinical trials
- TEa = 5 mmol/mol
- Bias = -0.5 mmol/mol
- SD = 1.5 mmol/mol
- SM = 3

Decision? Acceptable for routine testing, but not for clinical trials

3. QC planning

- QC strategies are established to reject performance when quality is less than least acceptable Sigma
- The difference between stable SM and SMa is the most tolerable shift; Critical Shift:

$\text{Shift}_{\text{crit}} = \text{SM} \rightarrow \text{SMa}$

 A certain shift results in more increased defects in a low sigma method than in a high Sigma method

Different SMs → Different critical shifts → Different QC strategies

SM-individualized QC plans



1. Systematic Error: Shift in Calibration



2. Random Error: Increase in Imprecision

Example

С

> The further from edge, the more safety margin (the safer)

T PB

> The more safety margin, the less surveillance

Α

B








Power Graphs: Tool for choosing appropriate QC



Power Graphs

Example

A1C in **routine** testing

SMa = 2

A. SM=7

B. SM=5

C. SM=3.5



Power Graphs Sigma Scale for SMa = 2 **SMa** 4.0 5.0 7.0 6.0 8.0 1.0 Example R 0.9 Ν for rejection (P) A1C in **Clinical Trials** 3s^{/2}2s^{/R} 0.8 0.03 SMa = **4** 0.7 0.01 0.6 13s/22s/R4s A. SM=7 0.01 2 0.5 '2s **B. SM=5** <20% 0.05 0.4 '3s **C. SM=3.5** 0.3 0.00 Prob 2.5s 0.2 0.01 0.1 '3s 0.00 ----0.0+ 0.0 1.0 2.0 3.0 4.0 Systematic error (SE, multiples of s)

Sigma Metric & Max E(Nuf) QC model

Optimizing Run Size based on Sigma



Planning Risk-based SQC Schedules for Bracketed Operation of Continuous Production Analyzers. Westgard JO, Bayat H, Westgard SA

Westgard Sigma Rules® with Run Sizes



SM-individualized QC:

Using SM for planning individualized QC Strategies N, R, Rules, Number of Rules, Run size

Establishing Evidence-Based Statistical Quality Control Practice. Westgard JO, Westgard SA.

Applications of Sigma Metric

4. EQAS Which for Routine testing? Figure 1 Assigned value Quality goal +/- 4,0 mmol/mol Which for Clinical Trials? 2017.10 72 What about QC? **Negative** 68 pullom 62.1 СТ **C**1 IFCC –WG A1C СТ TV: 61.1 CT 60 TEa = 5 mmol/mol 56.1 Abbott HbAle Variant Turbo/BioRad kal Tosch/Tosch ka Minion - HbAl APILLARYS, Sebi DCA Vantag fosoh/Mono S ku Tina-qua 10 Hemoglobin Al 00 Hemoglobin Al HPLC-Mono OCUe - HEA SMa: FOSONACA Routine = 2Ο Clinical Tr. = 4 Ο Accuracy of Hb A1c monitored by EQQ and compared with patient mean values. Gunnar Nurdin JDST. 2018, Vol. 12(4) 771-779

Question:

Applications of Sigma Metric

5. Improving Performance

Sigma Decision Charts

- Bias vs SD
- Operating Point



5. Improving Performance

Normalized Sigma Decision Charts:

- B/TEa% vs SD/TEa%
- Differentiae between the effects of bias and imprecision on SM
- Compare different performances
- Which influence factor (B or SD) must be decreased to improve performance



Normalized Sigma Decision Charts Comparison of Different APS



Shifts assumed in Six Sigma:

- the worst case shift is 1.5 SD
- revers by themselves

Shifts in Medical Laboratory:

- are not always less than 1.5 SD
- do not reverse by themselves
- we use **QC** to detect **critical** shifts and fix them



$\sigma_{\textbf{QC}}$

Shift necessary for QC to reach the intended Ped **Worst SM = SM - \sigma_{QC}**

Stable	Stable
SM	DR
6σ	0.002
5σ	0.3
3.5 σ	233



The more powerful is QC, the lower is the shift necessary to reach the intended Ped

1.5 SD shift in Medical Laboratory?

	Defect Rate		
	Stable		Worst Case
	State	1:3 s	1:3s, 2:2s, R4s, 4:1s
	(No	N1; R1	N=4, R=1
SM	Shift)	(σ _{QC} = 3.7)	(σ _{QC} = 2.2)
6 σ	0.002	10700	72
5σ	0.3	96800	2555
3.5 σ	233	508000	96800

- Even with the practically toughest QC, long-term DR is much more than expected in Six Sigma methodology (assuming 1.5 SD shift as the worst case)!

- Long-term Defect Rate is determined by the sensitivity of QC (i.e. σ_{oc}), not 1.5 SD shift

Example

Defect Rate (DPM)		
	Stable State	
6σ	0.0002	
4σ	1350	

right QC

Reaches the intended Ped at SE_{crit};

 $\sigma_{QC} = SE_{crit}$



Example

L S	, Defect (DPM)			
igm .eve	Stable		Worst Case	
ھ _	State	SMa = 2		
6σ	0.0002	22750		
4σ	32	22750		

With right QC, Long-term Defect Rate is determined by the least acceptable quality, i.e. SMa.



Short-term/Long-term: Equivalent terms in medical laboratory



> Degree of risk for patients is determined by the ability of QC

Expected long-term defect rate of analytical performance in the medical laboratory: Assured Sigma vs. observed Sigma

Biochem Med (Zagreb) 2018;28(2):**020101.** https://doi.org/10.11613/BM.2018.020101

Max allowable temperature



Short-term/Long-term: Practice in medical laboratory

Evaluate performance

Determine Bias & SD

- Calculate SM
- Is SM > SMa?
- If yes, Plan right QC strategy to assure SMa

FORGET 1.5 SD ASSUMED SHIFT!

Teşekkürler

